The Order of Operations

Objective:

1. Students will be able to write expressions that record calculations with numbers.
2. Students will be able to solve expressions by following the order of operations.

Overview:
In this lesson, students will gain exposure to the order of operations and its rationale, and will write, interpret, and evaluate such expressions. The focus of this lesson is on the interpretation and writing of expressions using parentheses and/or the order of operations, but teachers may want to include additional practice with the evaluation of such expressions.

Key Content Standard(s):
5.OAA.A.2

Key Practice Standard(s):
4

Lesson Plan:

1. Give students the following situation: “Nataly has $13. Her neighbor pays her $8 an hour to rake leaves. If she rakes for four hours, how much money does she now have?” Give students a moment to answer the question, and ask for students to explain how they did so.

2. Now ask students to model the situation. Insist that they model it exactly as is written (i.e. the first thing they write should be “13”). Give students a moment to do so. Ask students to share the answers they got. If students are not forthcoming with answers other than 45, ask if anyone got 84 or 136. Congratulate any such students on their honesty, and ask the class what happened? Why is it that when you write “13 + 8 x 4” the answer is 45 and not 84 (or 136 if the 4 is written first)?

3. Students who have seen parentheses before may say that 8 x 4 must be put in parentheses. Other students may say that you have to write the 8 x 4 first. Respond that both of these strategies will result in the answer 45, but neither is necessary. Ask students which operation must be done first to achieve a correct result of 45 (answer: 8 x 4). Tell students that in an expression with addition and multiplication, the multiplication is always done first. Tell students that Nataly’s situation demonstrates why this is so. You may give a few numerical examples for students to quickly practice for accuracy (e.g. 3 + 7 x 4 or 0.4 + 8.3 x 7).

4. Tell students these rules are true for subtraction and division as well as addition and multiplication. Therefore, when you see multiplication or division in an expression, you do that before addition or subtraction. Give students a few examples (e.g. 9 + 27 ÷ 3 – 2 x 4). Tell students that you first do all the multiplication and/or division (going left to right) before doing all the addition and/or subtraction (again going left to right).
5. Tell students that sometimes there are situations where you must do something first, regardless of what operation you are using. For example: Eric is playing a video game. At a certain point in the game, he has 31500 points. Then the following events happen, in order:

- He earns 2450 additional points.
- He loses 3310 points.
- The game ends, and his score doubles.

6. Ask students to model this situation. If students write 31500 + 2400 – 3310 x 2, ask students what you would do first, if you follow the order of operations. Students should recognize that you would do 3310 x 2 first. Remind them that Eric’s entire score doubled, not just 3310 (and in fact, he lost the 3310 points). Tell students that we need a way to show that certain operations must be done first, and we can do that using parentheses. Put parentheses in the correct place (e.g. (31500 + 2400 – 3310) x 2). Tell them that when an expression is written like this, we know we must do what it inside the parentheses first. You may want to give students a few numerical expressions using parentheses to evaluate [e.g. (9 – 4) x (8 + 3) x 2]. You should also show students expressions using brackets or braces, and them that these are also acceptable symbols that mean the same thing.

Assessment:

Have students respond to the following scenario:

Eric's sister Leila plays the same game. When she is finished playing, her score is given by the expression $3(24500+3610)–6780$.

Describe a sequence of events that might have led to Leila earning this score.

Differentiation:

Order of operations is often taught only with whole numbers, and does not to be so. The numerical examples could include decimal numbers or mixed numbers.

Commentary:

The point of this lesson is to guide students to being able to correctly model situations requiring more than one operation and/or requiring the use of parentheses. The introductory situation is quite easy for students to interpret in real-world terms – the vast majority will be able to recognize that she makes $32$
for raking the leaves, and that in addition to her $13, she now has $45. However, when students who
do not know how to consistently follow the order of operations model this expression, they will either a)
evaluate the expression incorrectly, or b) address the problem by simply writing $8 \times 4 + 13$ or $13 + (8 \times 4)$
without recognizing that the parentheses are unnecessary.

You may want to teach the popular acronym PEMDAS (parentheses, exponents, multiplication, division,
addition, subtraction), though it is not, strictly speaking, necessary. If you do teach it, take pains to
ensure that students know that multiplication and division “go together,” as do addition and
subtraction.

It is often a good idea when having students evaluate expressions using the order of operations to have
students write out each step. For example:

\[
\begin{align*}
9 + 27 ÷ 3 & - 2 \times 4 \\
9 + 9 & - 2 \times 4 \\
9 + 9 & - 8 \\
18 & - 8 \\
10 &
\end{align*}
\]

However, the point of this particular standard is not necessarily to have students evaluate expressions,
but for them to correctly interpret them.

If applicable, include worksheets, diagrams, student work etc. at end
with the same factors 
(e.g. 3 x 3 = 9)

(31500 + 2450 -3310) x 2

Her score was 24500.

She earns an additional

In the game called Machine Masters, Leila collects 24500 coins. A rich man finds her and gives her an additional 3610 coins. Since she unlocked an achievement, she earns a 3x multiplier. But a monster steals 6780 of her coins.
3150 + 2450 = 3310.2

\[(3150 + 2450 - 3310)^2 \]

\[(33950 - 3310)\]

30640.2

61280

Leila is playing a game. First she starts with 24500 points and then she gets 3610 points. At the end of her first round her score is triple, but she dies so her score goes down so she loses 61280 points.
Leila scored 24,500 points later on. She scored 3,610. Then a bolder crushed her open eye, so her score trippled. Then a bear and a snake were chasing her and ran her into a trapped cave where they ate her, so she lost 6,780 points.
Leila plays a game to elf attack.

Leila plays the same game for 20 mins and this is what happens:
1. She starts with $24500.
2. She earns $3610.
3. Halfway through the game, her money is tripled.
4. At the end, she says: 6740 for an entry.